APPLICATION OF EXTREMA OF MULTIVARIABLE FUNCTIONS IN ECONOMIC OPTIMIZATION PROBLEMS

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Abstract

This article analyzes the application of methods for finding extrema of multivariable functions in economic optimization problems. Issues such as minimizing production costs, maximizing profit, and efficient allocation of resources are studied through mathematical modeling. Conditional and unconditional extrema are determined using the Lagrange multiplier method and second-order sufficient conditions. Theoretical concepts are linked with practice through economic examples, such as two-product production and consumer choice problems.

Keywords: multivariable function, extremum, optimization, economic problem, Lagrange method, profit function, profit maximization, cost minimization, conditional extremum, derivative, Hessian matrix and determinant, economic model, mathematical modeling

In a modern economy, making effective decisions requires taking into account the interaction of various factors. A firm's profit depends not on a single variable but on several variables—such as production volume, resource prices, technological level, and others. Therefore, studying multivariable functions and finding their extrema forms the core mathematical apparatus of economic optimization problems.

The purpose of this article is to demonstrate methods for finding unconditional and conditional extrema of multivariable functions using economic examples, as well as to show how the type of extremum can be determined using the Hessian matrix and its determinant.

Theoretical Foundations of Extrema of Multivariable Functions

Unconditional Extremum

Suppose a function $f(x_1, x_2, \dots, x_n)$is given, depending on $n$variables. A necessary condition for finding a local maximum or minimum is:

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$$\frac{\partial f}{\partial x_i} = 0, i = 1,2, \dots, n$$

These conditions yield stationary points where the first-order partial derivatives are zero.

To determine whether a stationary point is a maximum or minimum, second-order sufficient conditions are used. For this purpose, the Hessian matrix is constructed:

$$H(f) = \begin{bmatrix} \frac{\partial^2 f}{\partial x_1^2} & \frac{\partial^2 f}{\partial x_1 \partial x_2} & \cdots \\ \frac{\partial^2 f}{\partial x_2 \partial x_1} & \frac{\partial^2 f}{\partial x_2^2} & \cdots \\ \vdots & \vdots & \ddots \end{bmatrix}$$

a) If the Hessian matrix is positive definite (all leading principal minors > 0), the point is a local minimum.

b) If it is negative definite (signs alternate, first minor < 0), the point is a local maximum.

c) If it is indefinite, the point is a saddle point.

Conditional Extremum (Lagrange Method)

In many economic problems, variables are subject to constraints—for example, budget constraints or production limits. In such cases, the Lagrange function is constructed:

$$L(x_1, \dots, x_n, \lambda) = f(x_1, \dots, x_n) + \lambda \cdot g(x_1, \dots, x_n)$$

where $g(x) = 0$is the constraint. Then, stationary points are found from:

$$\frac{\partial L}{\partial x_i} = 0, \frac{\partial L}{\partial \lambda} = 0$$

Economic Optimization Problems

Profit Maximization (Unconditional)

Assume a firm produces two types of products. The profit function is given by:

$$\pi(x, y) = -2x^2 - 3y^2 - 2xy + 100x + 120y$$

where $x$and $y$represent production volumes.

Step 1: First-order derivatives

$$\frac{\partial \pi}{\partial x} = -4x - 2y + 100 = 0$$
$$\frac{\partial \pi}{\partial y} = -6y - 2x + 120 = 0$$

Solving the system:

$$\begin{cases} 4x + 2y = 100 \\ 2x + 6y = 120 \end{cases}$$

we obtain:

$$x = 18, y = 14$$

Stationary point: $(18; 14)$

Step 2: Hessian matrix

$$H = \begin{bmatrix} -4 & -2 \\ -2 & -6 \end{bmatrix}$$

Determinant:

$$\det(H) = (-4)(-6) - (-2)(-2) = 20 > 0$$

Since the first principal minor is negative, the matrix is negative definite.
Therefore, the point $(18, 14)$is a local maximum.

Maximum profit:

$$\pi(18; 14) = 648 - 588 - 504 + 1800 + 1680 = 1740$$

Cost Minimization (Conditional Extremum)

A firm must produce $Q = 100$units using two inputs:

$$Q(L, K) = L^{0.5}K^{0.5}$$

where L is labor and K is capital. Input prices: $w = 4, r = 9$

Total cost:

$$C(L, K) = 4L + 9K$$

Constraint:

$$LK = 10000$$

Lagrangian:

$$L = 4L + 9K + \lambda(10000 - LK)$$

First-order conditions:

$$4 - \lambda K = 0$$
$$9 - \lambda L = 0$$
$$10000 - LK = 0$$

From ratios:

$$\frac{K}{L} = \frac{4}{9}$$

Substitute into constraint:

$$L \cdot \frac{4}{9} L = 10000 \Rightarrow L = 150$$
$$K = 66.67$$

Minimum cost:

$$C = 4 \cdot 150 + 9 \cdot 66.67 = 1200$$

Economic Significance of Mathematical Modeling

Methods for finding extrema of multivariable functions are widely used in economics:

1) Determining optimal production volumes using profit functions
2) Allocating resources efficiently through cost functions
3) Maximizing utility in consumer choice problems
4) Finding equilibrium points in macroeconomic models

The concept of derivatives reflects marginal changes, while the Hessian matrix determines the nature of extrema. This ensures scientifically grounded economic decision-making.

Conclusion

This article demonstrated how methods for finding unconditional and conditional extrema of multivariable functions can be applied to economic optimization problems. In the profit maximization example, a local maximum was identified using the Hessian matrix. In the cost minimization example, the Lagrange multiplier method was applied.

Mathematical modeling and economic models were shown to be essential tools for analyzing real economic processes.

In the future, these methods can be effectively applied to dynamic optimization problems, multicriteria models, and econometrics**.**

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