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METHODS FOR CALCULATING THE INVERSE MATRIX AND DEVELOPING A CORRESPONDING PROGRAM

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Annotatsiya

Ushbu ishda matritsaning teskarisini topish usullari ko`rib chiqilgan. Uni misollar yordamida keng yoritilgan. Teskari matritsa topish usullariga mos dastur yaratilgan. Uch xil holdagi natijalar taqqoslab ko`rsatilgan.

Аннотация

В данной статье рассматриваются методы нахождения обратной матрицы. На работе подробно приведены примеры. Создана программа, помогающая находить обратную матрицу. Сравниваются результаты для трех различных случаев.

Annotation

This article discusses methods for finding the inverse of a matrix. Detailed examples are provided. A program for finding the inverse of a matrix is created. Results for three different cases are compared.

This work presents a non-traditional method for finding the inverse of a matrix. The theoretical information related to the calculation process is also provided. The inverse matrix is solved using both the classical method and the non-traditional method, and their solutions are compared. The advantages of using this method for finding the inverse matrix are demonstrated. In addition, a program for calculating the inverse matrix is presented. The program shows the advantages of finding the inverse matrix even when the matrix elements are given arbitrarily.

We know that if the determinant of a square matrix **A** is not equal to zero, then the matrix **A** is called a non-singular matrix. Otherwise, the matrix is called a singular matrix. The correctness of the calculated inverse matrix can be verified using equality $\square \cdot \square^{-1} = \square$. Here, **E** is the identity matrix having the same dimension as matrix **A**.

For a non-singular matrix **A**, there exists a unique inverse matrix $\square^{-1}$, which can be calculated using the following formula:

$$\square^{-1} = \frac{1}{\det \square}\begin{pmatrix} \square_{11} & \square_{21} & \cdots & \square_{\square 1} \\ \square_{12} & \square_{22} & \cdots & \square_{\square 2} \\ \cdots & \cdots & \cdots & \cdots \\ \square_{1\square} & \square_{2\square} & \cdots & \square_{\square\square} \end{pmatrix}.$$

For matrix $\square = \begin{pmatrix} \square_{11} & \square_{12} & \square_{13} \\ \square_{21} & \square_{22} & \square_{23} \\ \square_{31} & \square_{32} & \square_{33} \end{pmatrix}$, we find the inverse matrix $\square^{-1}$using the classical method. For this purpose, we first calculate the determinant of matrix **A**. The determinant is calculated using the following formula:

$$|\square| = (\square_{11} * \square_{22} * \square_{33} + \square_{12} * \square_{23} * \square_{31} + \square_{13} * \square_{21} * \square_{32}) -$$
$$-(\square_{13} * \square_{22} * \square_{31} + \square_{12} * \square_{21} * \square_{33} + \square_{11} * \square_{23} * \square_{32})$$

After determining that the determinant is not equal to zero, it becomes possible to find the inverse of the given matrix. If $\det(\square) \neq 0$, then the algebraic cofactors are calculated using the following formulas:

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$$\square_{11} = (-1)^{1+1} * \begin{vmatrix} \square_{22} & \square_{23} \\ \square_{32} & \square_{33} \end{vmatrix} = \square_{22} * \square_{33} - \square_{23} * \square_{32}$$

$$\square_{12} = (-1)^{1+2} * \begin{vmatrix} \square_{21} & \square_{23} \\ \square_{31} & \square_{33} \end{vmatrix} = -(\square_{21} * \square_{33} - \square_{23} * \square_{31})$$

$$\square_{13} = (-1)^{1+3} * \begin{vmatrix} \square_{21} & \square_{22} \\ \square_{31} & \square_{32} \end{vmatrix} = \square_{21} * \square_{32} - \square_{22} * \square_{31}$$

$$\square_{21} = (-1)^{2+1} * \begin{vmatrix} \square_{12} & \square_{13} \\ \square_{32} & \square_{33} \end{vmatrix} = \square_{12} * \square_{13} - \square_{23} * \square_{32}$$

$$\square_{22} = (-1)^{2+2} * \begin{vmatrix} \square_{11} & \square_{13} \\ \square_{31} & \square_{33} \end{vmatrix} = \square_{11} * \square_{33} - \square_{13} * \square_{31}$$

$$\square_{23} = (-1)^{2+3} * \begin{vmatrix} \square_{11} & \square_{12} \\ \square_{31} & \square_{32} \end{vmatrix} = \square_{11} * \square_{32} - \square_{12} * \square_{31}$$

$$\square_{31} = (-1)^{3+1} * \begin{vmatrix} \square_{12} & \square_{13} \\ \square_{22} & \square_{23} \end{vmatrix} = \square_{12} * \square_{23} - \square_{13} * \square_{22}$$

$$\square_{32} = (-1)^{3+2} * \begin{vmatrix} \square_{11} & \square_{13} \\ \square_{21} & \square_{23} \end{vmatrix} = \square_{11} * \square_{23} - \square_{13} * \square_{21}$$

$$\square_{33} = (-1)^{3+3} * \begin{vmatrix} \square_{11} & \square_{12} \\ \square_{22} & \square_{22} \end{vmatrix} = \square_{11} * \square_{22} - \square_{12} * \square_{22}$$

If $\det(\square) \neq 0$, then we consider another method for finding the inverse matrix.

$$\square = \begin{pmatrix} \square_{11} & \square_{12} & \square_{13} \\ \square_{21} & \square_{22} & \square_{23} \\ \square_{31} & \square_{32} & \square_{33} \end{pmatrix}$$

In this method, to perform the calculation, the first two rows of the matrix are copied downward and written again, and then they are transferred to another matrix **B**.

$$\square = \begin{pmatrix} \square_{11} & \square_{12} & \square_{13} \\ \square_{21} & \square_{22} & \square_{23} \\ \square_{31} & \square_{32} & \square_{33} \\ \square_{11} & \square_{12} & \square_{13} \\ \square_{21} & \square_{22} & \square_{23} \end{pmatrix}$$

Then the first two columns are added to the right side.

$$\square = \begin{pmatrix} \square_{11} & \square_{12} & \square_{13} & \square_{11} & \square_{12} \\ \square_{21} & \square_{22} & \square_{23} & \square_{21} & \square_{22} \\ \square_{31} & \square_{32} & \square_{33} & \square_{31} & \square_{32} \\ \square_{11} & \square_{12} & \square_{13} & \square_{11} & \square_{12} \\ \square_{21} & \square_{22} & \square_{23} & \square_{21} & \square_{22} \end{pmatrix}$$

In the next step, the first row and the first column are removed. As a result, a convenient table is formed for calculating the 2×2 determinants.

$$\square = \begin{pmatrix} \square_{22} & \square_{23} & \square_{21} & \square_{22} \\ \square_{32} & \square_{33} & \square_{31} & \square_{32} \\ \square_{12} & \square_{13} & \square_{11} & \square_{12} \\ \square_{22} & \square_{23} & \square_{21} & \square_{22} \end{pmatrix}$$

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At this stage, the corresponding minors are determined for each element of the given 3 × 3 matrix. The 2 × 2 minors are calculated in the following order:

As a minor $\square_{\square\square}^{\square\square}$, we calculate the determinant of the 2 × 2 submatrix formed by the intersection of the □-th row and □-th row with the □-th column and □-th column of the matrix, and place the results in their corresponding positions.

$$\square_{12}^{12} = \begin{vmatrix} \square_{22} & \square_{23} \\ \square_{32} & \square_{33} \end{vmatrix} \qquad \square_{23}^{12} = \begin{vmatrix} \square_{23} & \square_{21} \\ \square_{33} & \square_{31} \end{vmatrix} \qquad \square_{34}^{12} = \begin{vmatrix} \square_{21} & \square_{22} \\ \square_{31} & \square_{32} \end{vmatrix}$$

$$\square_{12}^{23} = \begin{vmatrix} \square_{32} & \square_{33} \\ \square_{12} & \square_{13} \end{vmatrix} \qquad \square_{23}^{12} = \begin{vmatrix} \square_{33} & \square_{31} \\ \square_{13} & \square_{11} \end{vmatrix} \qquad \square_{34}^{12} = \begin{vmatrix} \square_{31} & \square_{32} \\ \square_{11} & \square_{12} \end{vmatrix}$$

$$\square_{12}^{34} = \begin{vmatrix} \square_{12} & \square_{13} \\ \square_{22} & \square_{23} \end{vmatrix} \qquad \square_{23}^{12} = \begin{vmatrix} \square_{13} & \square_{11} \\ \square_{23} & \square_{21} \end{vmatrix} \qquad \square_{34}^{12} = \begin{vmatrix} \square_{11} & \square_{12} \\ \square_{21} & \square_{22} \end{vmatrix}$$

$$\square = \begin{pmatrix} \begin{vmatrix} \square_{22} & \square_{23} \\ \square_{32} & \square_{33} \end{vmatrix} & \begin{vmatrix} \square_{23} & \square_{21} \\ \square_{33} & \square_{31} \end{vmatrix} & \begin{vmatrix} \square_{21} & \square_{22} \\ \square_{31} & \square_{32} \end{vmatrix} \\ \begin{vmatrix} \square_{32} & \square_{33} \\ \square_{12} & \square_{13} \end{vmatrix} & \begin{vmatrix} \square_{33} & \square_{31} \\ \square_{13} & \square_{11} \end{vmatrix} & \begin{vmatrix} \square_{31} & \square_{32} \\ \square_{11} & \square_{12} \end{vmatrix} \\ \begin{vmatrix} \square_{12} & \square_{13} \\ \square_{22} & \square_{23} \end{vmatrix} & \begin{vmatrix} \square_{13} & \square_{11} \\ \square_{23} & \square_{21} \end{vmatrix} & \begin{vmatrix} \square_{11} & \square_{12} \\ \square_{21} & \square_{22} \end{vmatrix} \end{pmatrix}$$

According to the theory of linear algebra, the inverse of an □-order square matrix is determined using the following formula:

$$\square^{-1} = \frac{1}{|\square|} \square^{\square}$$

Let us consider the following example. Here, it is required to find the inverse of the inverse of a 3 × 3 matrix.

$$\square = \begin{pmatrix} 3 & 1 & 2 \\ 6 & 5 & 4 \\ 7 & 8 & 9 \end{pmatrix}$$

In this problem, the inverse matrix is first found using the method of minors, and then the result is verified using the classical method with the help of algebraic cofactors.

For the given 3 × 3 determinant, the first and second columns are written to the right side of the determinant, and it is then calculated using Sarrus' rule. This rule applies only to 3 × 3 determinants and simplifies the process of calculating the determinant. We calculate the determinant corresponding to the given matrix as follows:

$$|\square| = \begin{vmatrix} 3 & 1 & 2 \\ 6 & 5 & 4 \\ 7 & 8 & 9 \end{vmatrix} = (135 + 96 + 28) - (70 + 96 + 54) = 39$$

Thus, $|\ \square\ | = 39 \neq 0$, which means that the matrix has an inverse.

In the initial step, the matrix is extended. To facilitate the calculation of minors, the first two rows of the matrix are copied and written below.

$$\begin{pmatrix} 3 & 1 & 2 \\ 6 & 5 & 4 \\ 7 & 8 & 9 \\ 3 & 1 & 2 \\ 6 & 5 & 4 \end{pmatrix}$$

Then, the first two columns are added to the right side.

$$\begin{pmatrix} 3 & 1 & 2 & 3 & 1 \\ 6 & 5 & 4 & 6 & 5 \\ 7 & 8 & 9 & 7 & 8 \\ 3 & 1 & 2 & 3 & 1 \\ 6 & 5 & 4 & 6 & 5 \end{pmatrix}$$

In the next step, the first row and the first column are removed, resulting in a convenient table for calculating the 2×2 determinants.

$$\begin{pmatrix} 5 & 4 & 6 & 5 \\ 8 & 9 & 7 & 8 \\ 1 & 2 & 3 & 1 \\ 5 & 4 & 6 & 5 \end{pmatrix}$$

In the next step, the 2×2 determinants are calculated, that is, the minors are determined. In general, the calculation of the minors is carried out as follows:

$$\square^{12}_{12} = \begin{vmatrix} 5 & 4 \\ 8 & 9 \end{vmatrix} = 45 - 32 = 13, \quad \square^{12}_{23} = \begin{vmatrix} 4 & 6 \\ 9 & 7 \end{vmatrix} = 28 - 54 = -26,$$

$$\square^{12}_{34} = \begin{vmatrix} 6 & 5 \\ 7 & 8 \end{vmatrix} = 48 - 35 = 13$$

$$\square^{23}_{12} = \begin{vmatrix} 8 & 9 \\ 1 & 2 \end{vmatrix} = 16 - 9 = 7, \quad \square^{23}_{23} = \begin{vmatrix} 9 & 7 \\ 2 & 3 \end{vmatrix} = 27 - 14 = 13, \quad \square^{23}_{34} = \begin{vmatrix} 7 & 8 \\ 3 & 1 \end{vmatrix} = 7 - 24 = -17$$

$$\square^{34}_{12} = \begin{vmatrix} 1 & 2 \\ 5 & 4 \end{vmatrix} = 4 - 10 = -6, \quad \square^{34}_{23} = \begin{vmatrix} 2 & 3 \\ 4 & 6 \end{vmatrix} = 12 - 12 = 0, \quad \square^{34}_{34} = \begin{vmatrix} 3 & 1 \\ 6 & 5 \end{vmatrix} = 15 - 6 = 9$$

As a result, the resulting matrix of the following 3×3 minors is obtained:

$$\square = \begin{pmatrix} 13 & -26 & 13 \\ 7 & 13 & -17 \\ -6 & 0 & 9 \end{pmatrix}$$

Using the calculated values, we compute the inverse matrix. By substituting the obtained values, the inverse of the given matrix can be determined.

$$\square^{-1} = \frac{1}{|\square|} \square^{\square}$$

We transpose matrix **B**:

$$\square^{\square} = \begin{pmatrix} 13 & 7 & -6 \\ -26 & 13 & 0 \\ 13 & -17 & 9 \end{pmatrix}$$

Based on the transposed matrix **B**, we calculate the inverse matrix.

$$\square^{-1} = \frac{1}{|\square|}\square^{\square} = \frac{1}{39}\begin{pmatrix} 13 & 7 & -6 \\ -26 & 13 & 0 \\ 13 & -17 & 9 \end{pmatrix}$$

Using algebraic cofactors, we verify whether the inverse matrix obtained by the classical method is correct. We know that in the classical method, the inverse of an $\square$-order square matrix is determined according to the following sequence. The general formula for finding the inverse matrix is as follows:

$$\square^{-1} = \frac{1}{|\square|}\begin{pmatrix} \square_{11} & \square_{21} & \square_{31} \\ \square_{12} & \square_{22} & \square_{32} \\ \square_{13} & \square_{23} & \square_{33} \end{pmatrix}$$

Here, the algebraic cofactor corresponding to the element $\square_{\square\square}$ is calculated as follows:

$$\square_{\square\square} = (-1)^{\square+\square} * \square_{\square\square}$$

By calculating the algebraic cofactors and placing them into the matrix, we can compute the result:

$$\square = \begin{pmatrix} 3 & 1 & 2 \\ 6 & 5 & 4 \\ 7 & 8 & 9 \end{pmatrix}, |\square| = 39$$

$\square_{11} = 45 - 32 = 2$ $\quad \square_{21} = -(9 - 16) = 7$ $\quad \square_{31} = 4 - 10 = -6$

$\square_{12} = -(54 - 28) = -26$ $\quad \square_{22} = 27 - 14 = 13$ $\quad \square_{32} = 12 - 12 = 0$

$\square_{13} = 48 - 35 = 13$ $\quad \square_{23} = -(24 - 7) = -17$ $\quad \square_{33} = 15 - 6 = 9$

We find the inverse matrix and compare it with the matrix obtained using the initial method, observing the consistency of the results.

$$\square^{-1} = \frac{1}{39}\begin{pmatrix} 13 & 7 & -6 \\ -26 & 13 & 0 \\ 13 & -17 & 9 \end{pmatrix}$$

The program code presented above calculates the inverse of a given 3×3 matrix based on the method of minors.

```
def print_matrix(M, title=""):
    if title:
        print(title)
    for row in M:
```

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```
        print("  ".join(f"{x:8.4f}" for x in row))
    print()


def determinant(A):
    n = len(A)

    if n == 1:
        return A[0][0]

    if n == 2:
        return A[0][0] * A[1][1] - A[0][1] * A[1][0]

    det = 0
    for j in range(n):
        minor = [row[:j] + row[j+1:] for row in A[1:]]
        det += ((-1) ** j) * A[0][j] * determinant(minor)

    return det


def cofactor_matrix(A):
    n = len(A)
    C = [[0] * n for _ in range(n)]

    for i in range(n):
        for j in range(n):
            minor = [
                row[:j] + row[j+1:]
                for k, row in enumerate(A) if k != i
            ]
            C[i][j] = ((-1) ** (i + j)) * determinant(minor)

    return C
```

```
def transpose(A):
    return [list(row) for row in zip(*A)]


def inverse_matrix(A):
    detA = determinant(A)

    if detA == 0:
        raise ValueError("If the determinant is equal to 0, the inverse matrix does not exist.")

    C = cofactor_matrix(A)
    adjA = transpose(C)

    n = len(A)
    invA = [[adjA[i][j] / detA for j in range(n)] for i in range(n)]

    return detA, invA


n = int(input("Enter the size of the matrix (n): "))

print("\nEnter the elements of matrix A (each row separately, with spaces between elements):")

A = []
for i in range(n):
    row = list(map(float, input(f"{i+1}-row: ").split()))

    if len(row) != n:
        raise ValueError("Each row must contain exactly n elements!")

    A.append(row)
```

```
print_matrix(A, "A matrix:")

detA, A_inv = inverse_matrix(A)

print(f"|A| = {detA}\n")
print_matrix(A_inv, "Inverse matrix:")
```

As a result of running the program code, the following values were obtained.

```
A matrix:
  3.0000    1.0000    2.0000
  6.0000    5.0000    4.0000
  7.0000    8.0000    9.0000

|A| = 39.0

Inverse matrixx:
  0.3333    0.1795   -0.1538
 -0.6667    0.3333   -0.0000
  0.3333   -0.4359    0.2308
```

The advantage of the non-traditional method for finding the inverse matrix lies in the fact that it allows one to easily compute the inverses of matrices with elements of varying values.

Several years of experience in teaching mathematical subjects to programming students have shown that guiding students to use programming while learning the topic significantly increases their interest in studying the material. At the same time, it noticeably improves the effectiveness of the lessons.

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