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COMPUTING CONFIDENCE IN CONCLUSIONS IN FUZZY RULE SYSTEMS IN CONCLUSION

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Introduction

Artificial intelligence (AI) methods are used as the main tool for decision-making in various information systems.

AI methods are fundamentally different from traditional statistical analysis methods, as they are designed to search for hidden patterns in databases and data warehouses and make decisions based on them.

Decision making is based on calculation methods, in which there is no mechanism for substantiating the results obtained, for example, decision-making in artificial neural network systems.

A significant drawback of intelligent systems is the lack of technologies to explain how this decision was made. Because of this, users have reasons not to trust the results. For example, in models of artificial neural networks. Here, to solve the problem of trust in the results obtained , studies on the transparency of artificial neural networks will be introduced, but in cases with multi-layer neural networks of large dimensions, the effect of studies is not very effective.

The problem of explaining the process of outputting results was put forward in the initial stages of developing intelligent systems. The explanation module constituted the basic structure of expert systems (ES). The purpose of this module is to make the ES "transparent" for the user, i.e. to provide the user with the ability to understand the logic of the ES actions, to give a reliable guarantee of the correctness of the results obtained.

The work [A.V. GUSEV] formulates four key principles for ensuring trust in conclusions in intelligent systems in medicine, through trust: in the manufacturer; in the data; in the models; in the product. It is necessary to provide an explanation of the operation of intelligent systems, as well as the availability of information about the algorithms used in them.

In practice, it is often necessary to test a hypothesis with incomplete or distorted information. Sometimes it is difficult to make accurate estimates, but despite the uncertainty from a third party, we make reasonable decisions. For ES to be useful, even in cases with a large number of uncertainties when forming deviations of parameters from the norm, they must also be able to

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do this. A classic example of a problem is medical diagnostics. There are always some doubts about the clarity of the manifestation of symptoms of a particular disease. Doubts about the presence of a specific disease in a patient remain, since the identified symptoms are present in other diseases.

The presence of omissions in the values of a number of attributes in the description of an object is not always a reason to refuse to make a decision on it. For example, when making a diagnosis and prescribing a course of treatment for a patient in medicine. Specialists may not have (is under repair) unique equipment for measurements or do not have a set of chemical reagents for express analysis of blood group.

To make decisions when feature values are missing, it is proposed to use a system with a fuzzy rule base, where each rule defines the function of object belonging to a class by a specific feature. Making decisions on a classified object with missing data in the object description is performed by calculating the resulting degree of confidence in the conclusion of fuzzy inference rules indicating the informativeness of features in the object description.

Statement of the problem

The recognition problem is considered in the standard setting. It is assumed that the set $E_0=\{S_1,\ldots,S_m\}$ of objects of 2 non-intersecting classes K_1 is given. And K_2. The description of objects is made with the help of n different types of features $X(n) = (x_1,\ldots, x_n)$, ξ of which are measured in interval scales, $n - \xi$ - in nominal. The numbers of quantitative and nominal features are designated I and J, respectively, $|I|+|J|=n$.

It is necessary to construct a fuzzy rule base for fuzzy classification inference using the confidence measure calculation method.

The process of forming a fuzzy rule base and an inference mechanism is implemented by sequentially performing the following steps:

- division into intervals of values of quantitative features according to the criterion of dominance of class representatives to determine the degree of confidence in the truth of a statement (membership function) in the conditional part of a fuzzy rule;
- conducting experiments on a subset of feature values, constructed with random deletion of feature values, with the aim of eliminating (covering) gaps between intervals;
- calculation of the membership function for nominal features in the classification;
- construction of a mechanism for fuzzy logical inference of classification.

1. Division into intervals based on the criterion of dominance of class representatives

A method is proposed for calculating non-intersecting intervals of quantitative features, within the boundaries of which the values of certain classes dominate. The boundaries of the intervals are calculated using the dynamic programming method based on the results of frequency analysis. According to the frequency of occurrence in each interval, the values of the feature in the description of objects of one of the classes dominate.

Let the value of the quantitative feature x_c in the description of the objects, the samples are ordered in non-descending order

$$r_1, r_2, \dots, r_m . \qquad (1)$$

According to the criterion defined below, an ordered sequence of the form (1) is divided into τ_c non-intersecting intervals $\left[r_{c_u}, r_{c_v}\right]^i$, $1 \le u, u \le v \le m$, $i = \overline{1, \tau_c}$.

Let be $d_t^i(u,v)$ the number of class representatives K_t in the interval $\left[r_{c_u}, r_{c_v}\right]^i$. The boundaries of the first interval $\left[r_{c_u}, r_{c_v}\right]^1$ on the sequence (1) are calculated by the maximum of the criterion

$$\left| \frac{d_1^i(u,v)}{|K_1|} - \frac{d_2^i(u,v)}{|K_2|} \right| \to \max . \quad (2)$$

The boundaries for are defined similarly for $\left[r_{c_u}, r_{c_v}\right]^p$, $p > 1$ the values of (1) not included in $\left[r_{c_u}, r_{c_v}\right]^1, \dots, \left[r_{c_u}, r_{c_v}\right]^{p-1}$. The procedure's stopping criterion is the coverage of all values of (1) by non-intersecting intervals. Fig. 1 shows the partitioning of the dominance intervals of the values c of the -th feature.

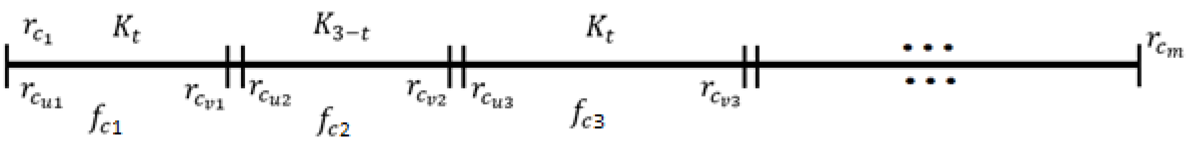

Fig. 1. Division into intervals of dominance of the values of *the c* -th feature.

Experts can use the division into intervals according to (2) when forming linguistic rules for knowledge bases. The number of intervals of class dominance indirectly indicates the status of regularities. The fewer the intervals of dominance, the stronger the manifestation of the regularity on a specific feature in the class. This property can be used when ranking quantitative indicators in applied problems. The highest ranks are received by those indicators whose number of intervals of class dominance is minimal.

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Quantitatively, dominance is expressed through the $f_{ci} \in (0.5,1]$ class membership function $K_l, l = 1,2$.

Let $\eta_{1i} = \frac{d_1^i(u,v)}{|K_1|}$ and be the frequency of occurrence of representatives $\eta_{2i} = \frac{d_2^i(u,v)}{|K_2|}$ and the number of representatives of the class, K_1respectively. AndK_2 in the interval $\left[r_{c_u}, r_{c_v}\right]^i$.

the value of the membership function $C$of the th feature to K_1 over the interval $\left[r_{c_u}, r_{c_v}\right]^i$ as

$$f_c(i) = \frac{\eta_{1i}}{\eta_{1i} + \eta_{2i}} . \qquad (3)$$

If the feature is nominal, then the membership function will have the form

$$f_c(a) = \frac{\mu_{1a}}{\mu_{1a} + \mu_{2a}},$$

where $\mu_{1a}, \mu_{2a}$is the number of gradation values$a$ features cin classesK_1 AndK_2 accordingly.

An important characteristic for data analysis, determined using the values of the membership function (3), is the stability of the feature.

Stability of the feature $x_c \in X(n), c \in I$:

$$U(c) = \frac{1}{p} \sum_{i=1}^{p} \begin{cases} f_c(i), & f_c(i) > 0,5; \\ 1 - f_c(i), & f_c(i) < 0,5. \end{cases}$$

Stability of the nominal feature $x_c \in X(n), c \in J$ by a set of gradation values $\mu \in \{1, \dots, p_c\}$:

$$U(c) = \frac{1}{m} \sum_{i=1}^{m} \begin{cases} f_c(\mu),\ x_{rc} = \mu\,, & f_c(\mu) > 0,5; \\ 1 - f_c(\mu),\ x_{rc} = \mu\,, & f_c(\mu) < 0,5; \\ 0,\ x_{rc} = \mu\,, & f_c(\mu) = 0,5. \end{cases} \qquad (4)$$

The indicator of the quality of partitioning into intervals is stability, the set of admissible values of which belongs to [0;1]. With the maximum value of stability$U(c) = 1$ each interval contains representatives (object values) of class K_1 or K_2.

The expediency of forming a sequence of features by measure $U(c)$is based on the assumption that the estimate (mathematical expectation) of the set of feature values according to (4) on training samples from the general population is unbiased. The results of a computational experiment on samples with a random distribution of gaps (unmeasured values) in the description of objects can serve as proof of the unbiased estimate.

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The feature $x_c \in X(n)$is stable if the values of the samples of the general population $U(c)$are equal to each other with an accuracy of *eps* > 0. Otherwise, the question arises about the adequacy of different samples in describing the problem by the feature x_c.

Let us denote by

$$w_1(t_1),\ldots,w_g(t_g),\ldots,w_n(t_n),\ t_i \in \{1,\ldots,n\} \quad (5)$$

sequence of values of stability of features, ordered in non-increasing order. The index of elements in (5) is interpreted as the rank of the feature. Let us determine the information content of a set of k features without gaps ($[n/2] \le k < n$) with respect to (5) in the description of an arbitrary admissible object S. Let $D(S,k)$ be the sum of the ranks of k measured values of features of the object S. The value of the sum is between $k(1+k)/2 \le D(S,k) \le k(2n-k+1)/2$.

To assess the informativeness of a set of k features of an object S, it is proposed to use the measure

$$\Omega(S,k) = 1 - \left(\frac{D(S,k)-\alpha}{\beta-\alpha}\right)\left(1-\frac{k}{n}\right), \quad (6)$$

where $\alpha = k(1+k)/2$, $\beta = k(2n-k+1)/2$.

The value of the information content assessment of a set of k features will lie in the range [0, 1] and can be interpreted as follows:

- [0, 0.6) – " *not sufficient* " ;
- [0 . 6, 0 . 72) – " *satisfactory* " ;
- [0 . 72, 0 . 86) – " *sufficient* " ;
- [0.86, 1.0] – " *appropriate* " .

The use of measure (6) is recommended when analyzing data in the description of arbitrary admissible objects.

2. Determining the resulting degree of confidence in the conclusion of fuzzy inference rules

In fuzzy logic, the degree of inclusion of a given element in a set is taken into account , which can continuously change in the range from 0 to 1. The specified degree of inclusion of an element in a set corresponds to the concept of the membership function of an element in a set .

The value of the membership function in the intervals of dominance of a quantitative feature *can* be used to construct a set of fuzzy rules for inference about the membership of an object S to the $K_l, l \in \{1,2\}$ species class

If A then B (< confidence coefficient >) , where < *confidence coefficient* > – the confidence coefficient (the value of the membership function) for the conclusion B if condition A *is met.*

Rules of inference for sets Iintervals for the featurex_c in the description of the object $S(x_1, \dots, x_n)$ has the form

$$P_c^k: \ E \ if \ x_c \in [r_{c_u}^k, r_{c_v}^k] \ thatS \ \in \ K_1 \ (f_c(k))$$

For each feature, a subset of inference rules is formed in accordance with the set of intervals constructed according to criterion (2).

For a nominal feature, c ∈ $I$the type of fuzzy rules is

$$P_c^a: \ Ifx_c = a \ ThatS \ \in \ K_1 \ (f_c(a)),$$

where $a$is the gradation of values of the nominal feature.

The fuzzy rule base for classification will consist of n groups of rules (a group of rules for each variable $x_c \in X(n), c \in I \cup J$).

For the object being classified, $S(x_1, \dots, x_n)$a subset of rules $\{P_i^k\}, i \le n$is triggered in the inference process , where $x_i \neq null$.

Thus, the problem of determining the resulting degree of truth of a conclusion based on the operation of subsets of rules (evidence) arises . It should be noted that the study of such problems relates more to the theory of evidence than to fuzzy logic.

A scheme using pairs of evidence to obtain a degree of confidence (a measure of trust) was proposed by Shortliff []. The formula for the measure of trust is as follows:

$$MD[h:e_1, e_2] = MD[h:e_1] + MD[h:e_2](1 - MD[h:e_1]), \quad (4)$$

Where h – hypothesis, e_1, e_2 - events confirming the hypothesis.

For the classification problem, formula (4) will be interpreted as follows

$$MD[S \ \in \ K_1: P_1, P_2] = MD \ [S \ \in \ K_1: P_1] + MD \ [S \ \in \ K_1: P_2](1 - MD \ [S \ \in \ K_1: P_1]),$$

where$P_1, P_2 \in \{P_i^k\}$.

The following certificates (for example, rules P_3) are taken into account in a cascade manner:

$$MD \ [S \ \in \ K_1: \text{МД}[S \ \in \ K_1: P_1, P_2], P_3].$$

Since the value of the quantitative feature in the description of the sample objects is discrete, i.e. they do not form a continuous sequence, there are gaps between the intervals. Uncertainty may arise in determining the degree of confidence of statements in the inference rules when the value of the featurex_c in the description of the object S falls into the gaps between the intervals.

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To cover the gaps between intervals, it is proposed to transfer the computational experiment on constructing an additional set of dominance intervals for (1) a feature $x_c$with random deletions of values in the description of sample objects E_0.

An algorithm for an additional set of dominance intervals is proposed:

1. Let $I = \{I_k\}_{k=1}^{l}$, Where $I_k = [r_{c_u}^k, r_{c_v}^k]$ – intervals constructed on (1) for the c-sign according to criterion (2), $l$is the number of intervals and $Z = \{z_i\}_{i=1}^{l-1}$, where $z_i = [r_{c_v}^i, r_{c_u}^{i+1}], i = \overline{1, l-1}$;
2. Let$II = \emptyset$;

3 . Bye$Z \neq \emptyset$ then do the following:

2.1. $R_s = R \setminus \{r_{c_1}^*, \dots, r_{c_p}^*\}$, where $r_{c_j}^*$are randomly selected members of the sequence (1), where $|U(R) - U(R_s)| < eps$;

2.2. Break $R_s$into intervals $I_R = \{I_k^*\}_{k=1}^{p}$according to criterion (2), where $I_k^* = [r_{c_u}^k, r_{c_v}^k]$;

2.3 $I_k^* \in I_R$. If $z_i \in Z$for exists , $I_k^* = z_i \cap I_k^*$ Then $I^* = I^* \cup I_k^*, Z = Z \setminus z_i$;

4 . The end.

The result of the algorithm's work can be illustrated in the following figure (Fig. 2).

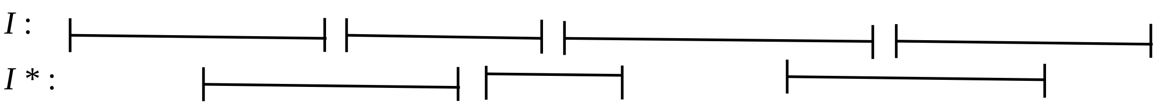

Fig. 2. Covering the gaps between intervals in I with set $I*$.

One of the advantages of systems based on fuzzy rules is the use of linguistic variables to draw conclusions in natural language. This approach is one of the options for implementing a linguistic processor in information systems for the output (explanation) of results in a limited natural language in terms of the subject area.

In most cases, the intervals of linguistic variables are set by experts, the type of membership function is selected from a given set (trapezoid, bell - shaped, etc.). In this work, it is known about intervals that their number is greater than or equal to 2. The exact number of intervals is determined by the algorithm using databases from the subject area. The value of the membership function is calculated based on the frequency of occurrence of representatives of the class of objects in the interval under consideration. It is possible to define a table of correspondence between the values of a linguistic variable and the intervals of values of the membership function of an object S to K_1 (table 1) .

Table 1. Correspondence of the values of the linguistic variable to the membership function

Meaning of the membership function $f_c(a)$	Meaning of linguistic variable
[0 .. 0, 2)	*practically absent*
[0.2 .. 0.35)	*small*
[0, 3 5.. 0,5)	*tangible*
[0.5..0.7)	*sensitive*
[0.7 .. 0.85)	*strong*
[0.85 .. 1]	*very strong*

Based on Table 1, the contribution of each feature x_c to the division of objects into classes can be expressed in natural language.

Description of the result of the system's work on classification of an object $S(a_1, \dots, a_n)$by a set of fuzzy rules $\{P_i^k\}, i \leq n$, where $a_i \neq null$, in natural language can be obtained as a synthesis of the values of linguistic variables of the rules involved in calculating the fuzzy *rules* (4). Trust in the results obtained can be determined by the measure of informativeness (7).

3. Computational experiment

For the experiment, a data sample " *E chocardiogram* " from [148] was taken , describing condition of patients who had a heart attack. The sample consists of 108 objects, of which 74 belong to the class K_1 (patient died within 1 year), 34 k K_2 (patient remained alive or died after 1 year).

Each object is described by a set $X(10) = (x_1, \dots, x_{10})$, containing 8 quantitative characteristics (x_1 =« *survival* » , x_2 =" *wall-motion-score* " , x_3 =" *epss* " , x_4 =« *age-at-heart-attack* » , x_5 =« *lvdd* » , x_6 =« *wall-motion-index* » , x_7 =" *fractional-shortening* " , x_8 =" *mult* ") and 2 nominal (x_9 =" *alive-at-1* " , x_{10} =" *pericardial-effusion* ").

Table 1. Characteristics and intervals of dominance of values of initial quantitative characteristics

Sign	Name of the feature	Boundaries of intervals [u , v] according to (2)	The meaning of the membership function (3)	Meaning of linguistic variable
	Survival	[0.25, 10]	0.05	*practically absent*

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x_1		[12, 57]	0.82	*strong*
		[5.5, 5.5]	1	*very strong*
x_2	*wall-motion-score*	[7.5, 8]	0.13	*practically absent*
		[9.0, 14.5]	0.68	*sensitive*
		[15.0, 39.0]	0.33	*small*
x_3	*Epss*	[0, 10.3]	0.69	*sensitive*
		[11, 40]	0.36	*tangible*
x_4	*age-at-heart-attack*	[35, 64]	0.63	
		[65, 86]	0.3	
x_5	*L vdd*	[2.32, 2.32]	1	
		[3.0, 3.0]	0	
		[3.1, 4.55]	0.72	
		[4.56, 6.73]	0.37	
		[6.74, 6.74]	1	
x_6	*wall-motion-index*	[1, 1.3]	0.71	
		[1.31, 3]	0.31	
x_7	*fractional shortening*	[0.01, 0.24]	0.37	
		[0.25, 0.61]	0.81	
x_8	*Mult*	[0.28, 0.57]	0.28	
		[0.59, 0.81]	0.6	
		[0.86, 0.93]	0.31	
		[0.93, 1]	0.87	

Dividing the values of quantitative features into intervals of dominance in the description of objects is another way to identify hidden patterns. Table 1 shows the number of intervals of dominance of quantitative features and their boundaries for the sample.

From Table 4.9 , we can identify relatively strongly expressed patterns in the belonging of objects to classes (in the particular case of k K_1) , for the *survival features* in the interval [12 , 57] with a membership function value of 0 . 82 and *fractional-shortening* in the interval [0 . 25 , 0 . 61] with a membership function value of 0 . 81 with the stability of objects in the intervals of 0 . 86 and 0 . 7, respectively. If the boundaries of the intervals coincide (for example, as for *wall-motion-score* – [5 . 5 , 5 . 5]), then objects with such a value require additional testing for anomalies.

"The object $S(a_1, \ldots, a_n)$belongs to the class "Patient died within 1 year" (K_1)with a confidence of 0.76 (strong).

The object description is missing the following attributes: Epss , L vdd and Mult .

The information content of the set of features in the description of an object Sis 0.7 (sufficient) .

Contribution of feature values to classification:

Survival =15, (strong);

W all-motion-score = 10 (sensitive) ;

Fractional-shortening = 0.5 (strong);

…

Conclusion

Thus, the combined confidence measure is higher than when each piece of evidence is taken separately. This is consistent with the result we expected, since several showed the same thing the direction of evidence reinforce each other. This set of rules has been successfully used in many intelligent systems, which has led to their widespread use in subsequent developments.

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